HUTP–96/A021
hep-th/9606070

Anomalous Currents in $SCFT_4$

A. Johansen[1]

Lyman Laboratory of Physics,
Harvard University, Cambridge, MA 02138, USA

ABSTRACT

We analyse[2] the critical behaviour of anomalous currents in N=1 four-dimensional supersymmetric gauge theories in the context of electric-magnetic duality. We show that the anomalous dimension of the Konishi superfield is related to the slope of the beta function at the critical point. We construct a duality map for the Konishi current in the minimal SQCD. As a byproduct we compute the slope of the beta function in the strong coupling regime. We note that the OPE of the stress tensor with itself does not close, but mixes with the Konishi operator. As a result in superconformal theories in four dimensions ($SCFT_4$) there are *two* central charges; they allow us to count both the vector multiplet and the matter multiplet effective degrees of freedom. Some applications to N=4 SYM are discussed.

[1] E-mail: johansen@string.harvard.edu

[2] Talk given at the Fourth International Conference on Supersymmetry SUSY96, May 29-June 1, University of Maryland, College Park.

1. Introduction

A recent insight into quantum field theory has been the discovery by Seiberg of the electric-magnetic duality in non-trivial N=1 superconformal theories in four dimensions that can be realized as infrared fixed points of ordinary N=1 supersymmetric theories [1].

In this talk we discuss some aspects of critical behaviour of anomalous axial currents in the context of the electric-magnetic duality in N=1 supersymmetric theories[3].

We demonstrate that although the anomaly for the *renormalized* Konishi current in an off-critical theory is proportional to the beta function, this anomalous current remains anomalous at the conformal fixed point, and plays an important role in quantum superconformal field theory in four dimensions ($SCFT_4$). More specifically we show that the two-point correlator of two Konishi superfields behaves like $< J(x)J(y) >\sim [\beta(g^2)/g^4]^2|x-y|^{-4}$ where $g = g(|x-y|)$ is the running gauge coupling constant at the scale $1/|x-y|$ and β is the Gell-Mann-Low function. Consequently in a theory with a non-trivial conformal fixed point this correlator has a power-like behaviour $\sim 1/|x-y|^{4+2\delta}$ at large distances, $|x-y| \to \infty$, where δ is given by

$$\delta = \beta'(\alpha_*) \tag{1.1}$$

where $\alpha_* = g_*^2/4\pi$ is the critical value of the gauge coupling constant. Eq. (1.1) agrees with an analogous property that holds in two dimensions [3]. The components of the J supermultiplet, in particular the axial current a_μ, become conformal operators with non-canonical conformal dimensions.

We discuss some applications of this result to Seiberg electric-magnetic duality [1]. We construct a duality map for the Konishi operator. Using the electric-magnetic duality [1] we argue that the conformal dimension of the electric Konishi current is equal to that of its magnetic counterpart in the dual formulation. As a result we find a relation between the slopes of the beta functions in these two formulations in the conformal window.

These features have their counterpart in the appearance of an additional operator Σ that mixes with the stress-energy tensor at the level of OPE's and plays a fundamental role in $SCFT_4$. In the perturbation theory, Σ coincides with the Konishi current. Consequently the OPE algebra in $SCFT_4$ is characterized by *two* central charges. We discuss some aspects of OPE algebra in N=4 supersymmetric Yang-Mills theory.

The conserved currents have of course their canonical dimensions. However, away from the fixed point, the anomalous axial current that enters the supercurrent superfield $J_{\alpha\dot\alpha}$ is actually a linear combination of a conserved R current and the Konishi current. Therefore it contains parts with different scaling behaviour. As follows from the fact that the conformal dimension of the Konishi current is larger then its canonical dimension, the Seiberg non-anomalous R current coincides with the current which enters $J_{\alpha\dot\alpha}$ in the infrared. This last fact was first established in ref. [4] by analysing operator equations for anomalous currents.

---

[3] This talk is based on the paper [2].

2. Scaling dimension of the Konishi current

In order to determine the renormalization factor of an anomalous current it is convenient to consider its matrix element in an external gauge field.

For definiteness we shall consider SUSY QCD with $SU(N_c)$ gauge group and N_f flavours of chiral superfields Q_i and $\tilde{Q}^i$, $i = 1, ..., N_f$, in the fundamental and anti-fundamental representations, respectively. One can define the Konishi superfield $J = \sum_i (\bar{Q}_i e^V Q_i + \tilde{Q}_i e^{-V} \bar{\tilde{Q}}_i)$ which is singlet under the flavour global $SU(N_f) \times SU(N_f)$ group. This current is anomalous.

By differentiating the 1PI effective action S_{eff} with respect to the bare coupling constant $1/[g_0^2]$ (in the notation of ref. [5]) in an appropriate kinematical regime [6] [7] one gets

$$< \mathrm{Tr}\, W^2 >= \frac{\beta(\alpha)}{\beta_1(\alpha)} \cdot \frac{\beta_1(\alpha_0)}{\beta(\alpha_0)} \cdot \frac{1}{1 - \alpha_0 N_c / 2\pi} \mathrm{Tr}\, W^2_{ext}, \tag{2.1}$$

where β stands for total NSVZ beta function [8], β_1 is the one-loop beta function, $\alpha = \alpha(k) = g(k)^2/4\pi$ is the effective coupling at the scale of external momenta, and $\alpha_0 = g_0^2/4\pi$ is the bare coupling. The factor $1/(1 - \alpha_0 N_c/2\pi)$ appears because of the one-loop form of the Wilsonian action [5].

The anomaly in the Konishi current [9] is proportional to the operator $\mathrm{Tr}\, W^2$. Hence, the renormalized matrix elements of the Konishi current read [10]

$$< \bar{D}^2 J_{ren} >= \frac{N_f}{2\pi^2} \cdot \frac{\beta(\alpha)}{\beta_1(\alpha)} W^2_{ext}. \tag{2.2}$$

This implies that the z factor for the Konishi superfield is proportional to $\beta(\alpha_0)/\beta_1(\alpha_0)$. To be precise, the total z factor is $z = (1 - \alpha_0 N_c/2\pi)\beta(\alpha_0)/\beta_1(\alpha_0)$. Alternatively this fact can be deduced by differentiating the effective action for the chiral superfields with respect to a bare Z_0 factor in front of the kinetic terms Q and $\tilde{Q}$ in the bare Lagrangian. Note that for N=1 abelian SUSY theories the invariance of $J(1-\gamma)$ (here γ stands for the anomalous dimension of the fundamental chiral supermultiplets) under the renormalization group flow has been first established in ref. [5].

Let us now consider the correlator of two Konishi currents in the electric formulation of the theory. By using the Callan-Symanzik equation one can show that

$$< J(x) J(y) >\sim \left(\beta(\alpha)/\beta_1(\alpha)\right)^2, \tag{2.3}$$

where $\alpha = g^2/4\pi$ is the coupling constant at the scale $1/|x - y|$.

We consider the large distance limit of this correlator. The factor $(\beta(\alpha)/\beta_1(\alpha))^2 \to 0$ at $|x - y| \to \infty$ because α goes to the critical value α_*. Near the critical point the beta function is supposed to have a simple zero, i.e. $\beta(\alpha) = \beta'(\alpha_*)(\alpha - \alpha_*)$. This means that the correlators are power-behaved at the critical point, i.e.

$$\alpha_* - \alpha = |x - y|^{-\beta'(\alpha_*)} \ \text{ at } \ |x - y| \to \infty. \tag{2.4}$$

Here α is taken at the scale $1/|x-y|$. Substituting this expression into eq. (2.3) we get

$$< J(x)J(y) >\sim |x-y|^{-6-2\beta'(\alpha_*)}. \qquad (2.5)$$

Let us now consider a non-asymptotically free theory with $N_f > 3N_c$. Assuming that such a theory flows into a free theory, which corresponds to $\alpha \to 0$, we see that the factor $\beta/\beta_1 \to 1$ in the infrared. The correlator of Konishi current at large distances has an integral power-like behaviour which implies that all currents have a canonical dimension as expected in a free theory.

3. Konishi current in the magnetic theory

We consider now the magnetic formulation of the theory [1], which has the $SU(N_f - N_c)$ gauge group, N_f flavours of dual quarks $(q^i, \tilde{q}_j)$ in the fundamental and anti-fundamental representations, respectively, and the meson field M^i_j in the $(N_f, \bar{N_f})$ representation of the flavour $SU(N_f)\times SU(N_f)$ group. In contrast to the electric formulation this theory has a superpotential $S = M^i_j q^i \tilde{q}_j$. Thus the magnetic theory has two coupling constants.

We discuss the determination of the magnetic counterpart of the Konishi current of the electric theory.

It is convenient to consider an extension of the electric theory which includes an additional chiral superfield X in the adjoint representation of the gauge group with a superpotential $S_{el} = \mathrm{Tr}X^3$. Such a theory has been analysed in ref. [11] and shown to flow to a non-trivial superconformal theory (for $N_c/2 < N_f < 2N_c$, see ref. [11]). On the magnetic side the theory includes the meson fields corresponding to $(M_j)^i_{\tilde{i}}; \quad j = 1, ..., k$. It also includes an additional chiral superfield Y in adjoint representation of the dual gauge group $SU(kN_f - N_c)$. The magnetic superpotential reads $S_{mag} = \tilde{s}\mathrm{Tr}Y^{k+1} + \sum_{j=1}^{k} t_j M_j \tilde{q} Y^{k-j} q$, where $\tilde{s}$ and t_j are coupling constants.

The strategy that we shall employ is to construct non-anomalous Konishi currents for the dual versions of *this* theory, including the fields X and Y, and subsequently, by introducing large mass terms for these fields, recover the original theory and corresponding currents in a low-energy limit.

In the electric theory we define a non-anomalous current which is a linear combination of the Konishi current and a current constructed from the field X (the conservation of this current is broken by the superpotential S_{el}). The corresponding non-anomalous current in the magnetic theory can be found by using holomorphicity of the Wilsonian action. It turns out to be a particular linear combination of the currents of the fields q, $\tilde{q}$, M and Y.

In order to determine the duality map for the Konishi operator in the original SQCD we add now a mass term $m\mathrm{Tr}\, X^2$ and, via integration over X (or, equivalently, by the Appelquist-Carazzone theorem) go back to the minimal theory with fields Q, $\tilde{Q}$ on the electric side. Also, by the duality map for chiral operators constructed in ref. [11] *at the*

critical point, Y will have a mass term $m\mathrm{Tr}\ Y^2$ and we will then reproduce the original theory in the low-energy limit with the fields q, $\tilde{q}$ and M on the magnetic side. Moreover, to reproduce precisely the initial theory, we have to choose a phase in which the gauge group is broken down to $SU(N_f - N_c)$. Note that the integration over the heavy field X (and Y on the magnetic side) leads to a non-critical minimal SQCD because, in particular, the coupling constants do not have their critical values, e.g. $\alpha = \alpha_\sharp \neq \alpha_*$, $\lambda = \lambda_\sharp$. The effective Konishi operators in this effective non-critical SQCD will be obtained by dropping out the heavy fields appearing in the non-anomalous Konishi operators of the Kutasov model. The parameter m is the only scale in the Kutasov theory and plays the rôle of UV cutoff in this resulting non-critical theory. The resulting Konishi operators should be thought of as bare operators defined at this scale. In order to describe their duality map in the critical SQCD we have to analyse their RG flow towards the infrared.

After dropping the fields X, Y, on the electric side the Konishi current J_{el} has its usual form, and is expected to flow in the infrared to the corresponding current in the critical minimal theory. In the magnetic formulation an analysis shows that the magnetic counterpart of the Konishi operator in the infrared (of the minimal SQCD) is a linear combination $J_{mag} = AJ_a + BJ_{sp}$, where the currents J_a and J_{sp} are not conserved due to an anomaly and the superpotential respectively. The values of A and B which are both non-zero, will not be important for us.

Note that our identification of the magnetic counterpart for the Konishi supermultiplet implies that the operator $\mathrm{Tr}\ W^2$ in the electric theory matches on the magnetic side with a linear combination of the operator $\mathrm{Tr}\ W^2$ and the superpotential, which is proportional to the superdivergence of the operator J_{mag}. Thus the duality map that we get for the anomaly multiplet seems to be different from that of ref. [12] where it has been suggested $\mathrm{Tr}\ W_{el}^2 = -\mathrm{Tr}\ W_{mag}^2$.

By analysing the matrix elements of the currents in the off-critical magnetic theory one can show that the z matrix of renormalization of the non-conserved currents J_a and J_{sp} has the entries proportional to linear combinations of the beta functions of the two coupling constants of the model. That means that the anomalous dimension of the magnetic Konishi current is given by the minimal eigenvalue β'_{min} of the 2×2 matrix of the slopes of the beta functions. Combining this result with that for the electric theory we get

$$(\beta'(\alpha_*))_{electric} = (\beta'_{min}(\alpha_*, \lambda_*))_{magnetic}, \tag{3.1}$$

where α_* and λ_* are the critical values of the gauge and superpotential coupling constants, respectively. It is interesting to consider the strong coupling regime $(2N_f - 3N_c)/N_c << 1$. In this case the magnetic theory is weakly coupled, we can compute β'_{min} in perturbation theory, and thus we can explicitly obtain the slope of the beta function in the strongly coupled electric theory

$$(\beta'(\alpha_*))_{electric} = \frac{28}{3}\left(\frac{3}{2} - \frac{N_f}{N_c}\right)^2. \tag{3.2}$$

In the next section we collect some simple observations about $SCFT_4$ that were stimulated by the investigation carried out so far. We shall see that the Konishi current plays an important role in $SCFT_4$.

4. OPE algebra

We now study $SCFT_4$ at the level of OPE's, and not simply at the level of the classical conformal group. It turns out that:

i) the OPE of the stress-energy tensor $T_{\mu\nu}$ with itself does not close [4]; another operator Σ is brought into the algebra;

ii) there are *two* central charges, c and c', one related to $T_{\mu\nu}$, the other one to Σ;

iii) in general, Σ has an anomalous dimension; it coincides with the Konishi current J in the perturbation theory.

More precisely in an N=1 superconformal theory we have

$$\begin{aligned} J_{\alpha\dot\alpha}(z)J_{\beta\dot\beta}(z') =& c\frac{X_{\alpha\dot\alpha\beta\dot\beta}}{(s^2\bar s^2)^{\frac{3}{2}}} + \Sigma(z')\frac{Y_{\alpha\dot\alpha\beta\dot\beta}}{(s^2\bar s^2)^{2-\frac{h}{2}}} + \cdots, \\ \Sigma(z)\Sigma(z') =& \frac{c'}{s^{2+h}\bar s^{2+h}} + \cdots, \end{aligned} \tag{4.1}$$

where $J_{\alpha\dot\alpha}$ is the supercurrent, h stands for an anomalous dimension of the scalar (non-chiral) superfield Σ, $X_{\alpha\dot\alpha\beta\dot\beta}$ and $Y_{\alpha\dot\alpha\beta\dot\beta}$ are appropriate dimensionless tensor structures, $s_{\alpha\dot\alpha} = (x-x')_{\alpha\dot\alpha} + \frac{i}{2}[\theta_\alpha(\bar\theta-\bar\theta')_{\dot\alpha} + \bar\theta'_{\dot\alpha}(\theta-\theta')_\alpha] \bar s_{\alpha\dot\alpha} = (x-x')_{\alpha\dot\alpha} + \frac{i}{2}[\bar\theta_{\dot\alpha}(\theta-\theta')_\alpha + \theta'_\alpha(\bar\theta-\bar\theta')_{\dot\alpha}]$, we use the conventions of *Superspace* [14].

For an N=1 free theory with m matter multiplets and v vector multiplets, we have $c = 15v + 5m$, $c' = 5m$. For a generic interacting $SCFT_4$, c and c' encode, via these formulae, the effective numbers of matter and vector multiplets, m and v.

Consider now some applications to the N=4 supersymmetric Yang-Mills theory which is conformal for any value of the gauge coupling constant α. Since c is related to the conformal anomaly in an external gravitational field [15], [16] (which obviously does not have any multiloop corrections in this theory) it does not depend on α. In the weak coupling regime $c' = 15n$, $n = v = m/3$. It is not clear whether c' is also unaffected by turning on the interaction. S-duality tells us that $c' = 15n$ also in the infinitely strong coupling regime. It would be interesting to identify Σ in the interacting theory, and compute the corresponding c'. Note that a deformation Σ cannot preserve N=4 supersymmetry, since the only candidate in that case would be $\Sigma = \mathcal{L}$, which is not the case in the free theory. Thus, we expect Σ to have anomalous dimension $h \neq 0$. Indeed in the weak coupling limit an explicit computation (using Σ equal to the Konishi operator J) gives (for $SU(N_c)$ gauge group) $h = 3\alpha N_c/\pi$. In general due to S duality the parameters h and c' must be

[4] OPE's in dimensions $2 < d < 4$ have been studied recently by A. Petkou in ref. [13].

real non-singular modular functions of $\tau = \theta/2\pi + 4\pi i/g^2$; θ is a theta angle in front of $F\tilde{F}$ in the Lagrangian (more precisely, τ belongs to the fundamental domain of the $\Gamma_0(N_c)$ subgroup of $\mathrm{SL}(2,\mathbf{Z})$ [17]). A dependence of c' on the coupling constant of N=4 theory would imply that the effective numbers of chiral and vector multiplets (in the N=1 sense) changes with g while the total number does not [18].

The fact that the parameter h controls the scaling properties of the chiral superfields suggests the following interpretation of the model. The N=4 SYM theory may be viewed as a Wess-Zumino model coupled to an N=1 SYM. One may be interested in the effective Wess-Zumino coupling constant "dressed" by the gauge interactions. With such an interpretation h plays a role of the "dressed" anomalous dimension of the Konishi current in the Wess-Zumino model which is related to the effective β-function as $h = \beta' - \beta/\lambda$, where λ stands for the effective superpotential coupling constant [18]. This situation seems to be analogous to the gravitational dressing of the β-function in 2D gravity [19].

In conclusion, while the quantum conformal algebra can provide us, through the identification of two SCFT_4's with a map between the two operators Σ_{el} and Σ_m, the considerations of the previous sections show how to map the Konishi currents. It is easy to see that Σ is related to or even coincides with the Konishi operator (at least in the perturbation theory). The appearance of one anomalous operator in the OPE of the stress-energy tensor helps to clarify the role of anomalous currents in CFT_4.

Acknowledgements. The author is grateful to D. Anselmi and M Grisaru for collaboration, and to D. Freedman for interesting and valuable discussions. This work is partially supported by the Packard Foundation and by NSF grant PHY-92-18167.

References

[1] N. Seiberg, Nucl. Phys. **B 435** (1995) 129.

[2] D. Anselmi, M. Grisaru and A. Johansen *A Critical Behaviour of Anomalous Currents, Electric-Magnetic Universality and CFT*$_4$. Preprint HUTP–95/A048, BRX-TH-388; hep-th/9601023.

[3] A. Cappelli and J. Latorre, Nucl. Phys. **B 340** (1990) 659.

[4] I. Kogan, M. Shifman and A. Vainshtein, *Matching Conditions and Duality in N=1 SUSY Gauge Theories in the Conformal Window*. Preprint TPI-MINN-95/18-T, UMN-TH-1350-95, OUTP-95-25P, hep-th/9507170.

[5] M. Shifman and A.Vainshtein, Nucl. Phys. **B 277** (1986) 456.

[6] A. Johansen, Nucl. Phys. **B 376** (1992) 432.

[7] M.A. Shifman and A.I. Vainshtein, Nucl. Phys. **B 365** (1990) 312.

[8] V. Novikov, M. Shifman, A. Vainshtein and V. Zakharov, Nucl. Phys. **B 229** (1983) 407; *ibid* **B 229** (1983) 381; Phys. Lett. **B166** (1986) 329.

[9] K. Konishi, Phys. Lett. **B 135** (1984) 439.

[10] A.A. Anselm and A.A. Johansen, Zh. Eksp. Teor. Fiz. Letters **49** (1989) 185; Zh. Eksp. Teor. Fiz. **96** (1989) 1181.

[11] D. Kutasov, Phys. Lett. **351B** (1995) 230; D. Kutasov and A. Schwimmer, Phys. Lett. **354B** (1995) 315; D. Kutasov, N. Seiberg and A. Schwimmer, *Chiral Rings, Singularity Theory and Electric-Magnetic Duality*. EFI-95-68, WIS/95/27, RU-95-75; hep-th/9510222.

[12] See for a review K. Intriligator and N. Seiberg, *Lectures on Supersymmetric Gauge Theories and Electro-Magnetic Duality*. Preprint RU-95-48, IASSNS-HEP-95/70, hep-th/9509066.

[13] A. Petkou, Phys. Lett. **B 359** (1995)101 and hepth/9602054.

[14] S.J. Gates, M.T. Grisaru, M. Roček and W. Siegel, *Superspace*, Benjamin-Cummings, Reading MA, 1983.

[15] A. Cappelli, D. Friedan and J. Latorre, Nucl. Phys. **B 352** (1991) 616.

[16] See, for example, F. Bastianelli, Tests for C-theorems in 4D, hep-th/9511065, and references therein.

[17] C. Vafa and E. Witten, Nucl. Phys. **B 431** (1994) 3.

[18] D. Anselmi, D. Freedman, M Grisaru and A. Johansen, in progress.

[19] See, for example, M. Grisaru and D. Zanon, Phys. Lett. **B 353** (1995) 64, and references therein.